EPISTEMOLOGY, ONTOLOGY, AND METHOD: COMMENTS ON TIBERGHIEN'S AND DREYFUS' PAPERS

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I thank the conference organizers for giving me the opportunity to comment on two thoughtful, important papers. In the following I comment on the strengths of each paper, but focus more on providing constructive feedback on the stances Tiberghien and Dreyfus take. (More for this intro in the revised version.)

Tiberghien

Andrée Tiberghien shares a method for analyzing classroom discourse so as to gain insight into students' construction of knowledge from instruction. In the course of sharing her method Tiberghien makes several important distinctions that are necessitated by taking seriously the commonly accepted stance that *the knowledge one teaches is not necessarily the knowledge students learn*. In other words, her analytic method distinguishes between what is taught and what is learned in the same way that we are compelled to say about human communication that *what one says is not necessarily what others hear*.

But the distinction between taught knowledge and learned knowledge raises knotty problems regarding the ontology of knowledge. There are two ways in which we can understand Tiberghien's distinction, depending on where one imagines that knowledge exists.[1] One ontology of knowledge (ONT1) is to imagine that students and teachers in communicative interaction each have their personal understandings of what is being discussed and that they interact on the basis of those. There are more, subtle, issues that come with this view (e.g., see Steffe & Thompson, 2000; Thompson, 2000) but for the moment this specification is good enough. From the perspective of ONT1, *taught knowledge* exists within the researcher's understanding of a teacher's instructional actions and their impact on students. It is what the researcher understands from the teachers' and students' actions. The second ontology of knowledge (ONT2) is to imagine that knowledge exists in the interactions among the conversation's participants. I take ONT2 to express an emergent perspective on knowledge, that a group's knowledge is fundamentally different from a list of what each of the participants knows. One could call ONT2 the *Gaia* perspective, from Isaac Asimov's *Foundation* trilogy, where every living organism on the planet Gaia participated in one total mental functioning of the living organism named *Gaia*.[2]

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[1] There is a third option (ONT3), where perfect knowledge exists in a Platonic realm of ideal forms. Tiberghien seems to appeal to the third option when she speaks of "knowledge to be taught," which, it seems, is what an ideal observer would understand as the learning goals that students ideally should attain. The idea of "knowledge to be taught" is undeveloped in this presentation of her analytic method, but it is consistent with another ONT3-like statement about conventional knowledge: "… there is conventional knowledge that is independent of time in the sense that one can say this sentence means that and that" (p. 3).

[2] I am aware of the *Gaia Hypothesis*, which posits that Earth's living organisms actually form this unity, which, according to Wikepedia, was developed by James Lovelock in the

It seems, at times, that Tiberghien wants to have it both ways. On the one hand, she seems to rely on an ONT1 perspective when explicating her idea of *taught knowledge*.

> [Taught knowledge] is a researcher's construct based on the discursive productions of the class (including gestures); it corresponds to the knowledge staged in the classroom by the teacher and the students during a teaching sequence. (p. 2)

Taught knowledge is the sense a researcher makes of his observations of the interactions among students and teacher. Taught knowledge is the mathematics that emerges within the researcher's experience of those interactions. I stress here that "the mathematics that emerges" is entirely within the researcher's understanding. But Tiberghien's very next sentence is, "This taught knowledge is a joint production of the teacher and the students" (p. 2). This sentence, now about teachers and students participating in the production of this knowledge, has overtones of an ONT2 perspective. So, within this one stance we see two competing ontologies of taught knowledge—it exists within the researcher's understanding and it exists in the (actual) interactions of students and teachers.[3]

The confounding of ONT1 and ONT2 perspectives in Tiberghien's framework has repercussions for the ways she deploys her analytic method. For example, she wishes to investigate learners' construction of knowledge (p. 4), and does so in a way that adheres to her (very useful) theoretical stance that students' coming to understand relationships between ideas in physics "involves simultaneously … the relation [between ideas] *and* each term of it; it does not follow the rational decomposition of disciplinary knowledge" (p. 4). Tiberghien employs ONT2 perspectives when looking at long (10-minute) segments of instruction, and an ONT1-like perspective when looking at smaller (10-second) segments of instruction. To capture knowledge as emergent both within and between subjects, Tiberghien introduces the idea of facets, or propositional representations of knowledge (e.g., "if something is moving, a force is being applied to it"). Unlike Jim Minstrell, Tiberghien does not attribute facets to students. Rather, facets exist explicitly within a researcher's thinking. They are the researcher's construction. The researcher may use them to point to things a student might know, but Tiberghien makes a strong distinction between students' knowledge and researchers' facets (p. 7).

Tiberghien's use of facets in her analytic method provides a considerable amount of flexibility in thinking about students' thinking before making strong claims about what they might know or understand. But at the same time, one must, at some point, make claims about what students know. At that moment, facets, as described by Tiberghien, cease doing the work she needs done. If facets are never imputed to students, and are instead used only by the researcher as an aide to the researcher's thinking, then the

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1960's. However, Asimov's original *Foundation* series, including his description of *Gaia*, was written in the 1940's and 1950's.

[3] We also see overtones of ONT2 in Tiberghien's statement (p. 1) that "knowledge 'lives' within groups of people (a class in our case)."

researcher needs another representational system to speak about what students might know.

Tiberghien seems to actually use facets both as researcher constructs and as students' ways of thinking, that is both in the way she says she uses them *and* in the way Minstrell uses them. Her example (pp 8-9) of two students discussing forces applied during the toss and catch of a beach ball seems to exemplify this. The list of facets (researcher's constructs that emerge from the researcher's observations) later becomes students' ways of thinking. This is a methodological double play.[4] It is like using the phrase "taken as shared" to refer to something that emerges in one's observations of a group's behavior (it is *as if* the group shares such and such meanings) and then, later, using that inference to talk about individual students' meanings (Thompson, 2001; Thompson & Cobb, 1998). In my way of thinking, Tiberghien needs a theory that incorporates both student thinking and human communication to answer the question, "What are students' ways of thinking that might lead them to interpret each other and the teacher so that their collective behavior suggests these collective ways of thinking?"

In short, Tiberghien's attempt, which I applaud, to account for students' learning in instructional situations by distinguishing among "knowledge to be taught," "taught knowledge," and "learned knowledge" begs the question of how, theoretically, it is possible for these forms of knowledge to come into contact when they exist in different ontological spaces. One approach that cuts this Gordian knot is to attribute "knowledge" only to autopoietic systems, systems that are self-regulating, closed in their organization, yet whose components are open—meaning, the system continually regenerates itself by getting new parts (e.g., cells), but it continues to work the same way (Maturana, 1978; Maturana & Verela, 1980; Varela, Maturana, & Uribe, 1974). In this way, researchers, students, and teachers are knowing agents, but a class is not. A class persists for 45 to 90 minutes per day and then it disbands. Even then, if a system is indeed autopoietic, the nature of its organization and the interaction of its components may mean that different levels of organization have different kinds of knowledge. Thus, the double play mentioned earlier (attributing group knowledge to individuals composing it or vice versa) is an illegal move even if the group can be thought of as autopoietic.

If one takes seriously the notion that knowledge can be attributed only to autopoietic systems, then one must continually say to whom one is attributing knowledge when speaking of it. Taking this approach, of always stating to what knowing system the researcher is imputing knowledge, led Les Steffe and me to distinguish between accounts given by first-order observers and accounts given by second-order observers (Steffe, Glasersfeld, Richards, & Cobb, 1983; Steffe & Thompson, 2000; Steffe, Thompson, & Richards, 1982) and to a re-definition of teaching so that it would be consistent with a

[4] This is a play on Fenstermacher's (1978) notion of a methodological triple play. He claimed that researchers' of teacher effectiveness were taking their answer to the question, "What actions seem to lead to effective teaching?" as also answering the questions, "What defines effective teaching?" and "What should teachers do to teach effectively?"

perspective that classroom instruction involves interactions among autopoietic systems (Thompson, 1979, 2000). [5]

Dreyfus

Tommy Dreyfus (and his colleagues) point out the centrality of abstracting (and, by implication, generalizing) to significant mathematical learning. The Dreyfus group (hereafter, DG) is one of just a few research programs that take abstracting as a central goal, and hence as a central problem, of mathematics learning. While I will raise questions and issues about their approach and their theory, they are raised within a background of agreement on the importance of their work.

If I understand DG correctly, they have distilled the activity of abstracting to four central processes – *recognizing*, *building-with*, *constructing*, and *consolidating.* The issues I raise all center around whether these central constructs carry enough meaning to provide an appropriate grain size for the tasks of mathematics education research. I will argue that they do not. I will also raise issues having to do with the connection of RBC+C with related work, the distinction between describing versus explaining, creating ad hoc explanations versus designing for outcomes, and connections of their work to activity theory.

Grain size

Suppose we have two competing theories, A and B. Suppose they are both employed to describe events in the same videotape. If Theory A captures distinctions that Theory B does not, and we deem those distinctions to be important, then Theory B has a grain size that is too large for the phenomena we are working with.

Here is an example that suggests that RBC+C has a grain size that is too large. In a current 9th-grade teaching experiment with students studying algebra, they were asked to use a computer graphing program to define a function to model a half dozen situations that were structurally similar to this:

> I am traveling at a constant speed of 0.53 mi/hr. 5.2 minutes after I first looked at my watch I was 7.3 miles from home. (a) How far from home was I when I first looked at my watch? (b) Define a function whose graph shows how far from home I was at every moment during the 8 minutes after I first looked at my watch.

Students were encouraged to type expressions in open form so that they could have the computer do messy calculations for them.[6]

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[5] In my presentation, if there is time, I will mention an approach that draws on theories of intersubjectivity from a radical constructivist framework that, I believe, avoid the pitfalls mentioned here. But the constraint on how one can generate warrantable claims creates very cumbersome analytic methods.

After discussing the first two problems, a discussion that the teacher leveraged to talk about similarities in ways of thinking about the problems, students worked the remaining four problems in pairs. We then gave them this problem.

> I am thinking about a function that has a constant rate of change that for now I will call r and its graph passes through a point that for now I will call (x_1,y_1). In 15 minutes I will tell you what r, x_1, and y_1 are. Right now I want you to define a function that will pass through that point with a rate of change of r, so that the graph will be correct as soon as you type values for x_1 and for y_1.

We saw students take, in principle, two approaches to this problem. Most students looked at the patterns in what they had typed for the previous six problems. They then typed "x_1" wherever before they had typed an *x*-coordinate, they type "y_1" wherever they had typed a *y*-coordinate, and they typed "r" wherever they had typed a rate of change. A smaller number of students used a different approach. They re-employed their original reasoning, saying things like, "If you change x by $–x_1$, then you'll change y by $r(-x_1)$. So if the graph passes through (x_1,y_1) with a rate of change of r, then changing x by $–x_1$ to make *x* 0 will make y be $y_1 + r(-x_1)$. Both groups ended with the function defined as $y = rx + (y_1 + r(-x_1))$. But they did so for very different reasons. Students using the first approach employed what Piaget called pseudo-empirical abstraction – generalizing from patterns they saw in their written solutions. Students using the latter approach engaged in what Piaget called reflecting abstraction – they generalized from patterns they saw in their reasoning. In the first case, students' objects of reflection were the products of their reasoning – the written solutions. In the second case, they reflected upon their acts of reasoning, *not* on what they had written. In the first case, the written generalization *is* the generalization that students made. In the second case, the written generalization is an expression of their generalized thinking.

The significance of this example to me is that while Piaget's theory of abstraction (actually, theory of reflection) is able to distinguish between the two approaches, I am unable to see how RBC+C does distinguish between them. I think you agree that the second abstraction is more propitious for students' later mathematical learning than is the former. If RBC+C cannot, in principle, capture distinctions like this, then there is a flaw in the theory.

Instructional design

DG seem to have a theory of abstraction without a theory of thinking. This is not a pedantic point. To me, the litmus test of math ed theories is whether they help in the task of designing instruction so that it supports students in learning what we hope they learn. Since abstracting and generalizing are core mathematical processes, and since "to abstract" and "to generalize" are special cases of "to think", it seems to me a major

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[6] For example, in the present case they could type (7.3 + 0.53(-5.2)) to answer part (a), and y = 0.53x + (7 + 0.53(-5.2)) for part (b). The computer would do the arithmetic.

shortcoming of the RBC+C theory of abstraction that it does not address mathematical thinking or link with theories of how students learn specific concepts (including those that arise through abstracting and generalizing). (Example forthcoming)

Connections

What will I gain by adopting the RBC+C model over my current reliance on Piaget's constellation of constructs that fall under "reflecting abstraction"? Each explanation of learning/abstracting/generalizing that I've seen from the RBC+C model seems amenable to being explained by constructs entailed in reflecting abstraction, and, as illustrated above, reflecting abstraction has greater sensitivity to students' contexts (i.e., how they are thinking). DG does not help me answer this question, because to date they have not compared their model to Piaget's and therefore have not addressed the twin questions of: (a) What is lacking in Piaget's theory that RBC+C provides; (b) What is lacking in RBC+C that Piaget's theory provides? I have ideas about this, but DG should be the ones who make this comparison.

Activity theory

Finally, I should point out that Dreyfus' paper does not make clear why they chose Activity Theory as their foundational way of thinking about actions. However, in (Herschkovitz, Schwartz, & Dreyfus, 2001) they do explain this. In their telling, activity theory did not give them a theory of abstraction. Rather, Davydov's account of activity theory gave them, in their opinion, a useful starting point. Davydov did not treat abstraction as separated from context, but as embedded in concrete human activity.

> Abstraction starts from an initial, simple, undeveloped first form, which need not be internally and externally consistent. The development of abstraction proceeds from analysis, at the initial stage of the abstraction, to synthesis. It ends with a consistent and elaborate final form. It does not lead from concrete to abstract but from an undeveloped to a developed form of abstract in which new features of the concrete are emphasized.

In other words, the activity itself can be the initial, simple, undeveloped first form of the abstraction that will come. This is an appealing position because, as instructional designers, everything we ask students to do can be leveraged with regard to processes of abstraction and generaliztion. However, as I noted already, without a theory of mathematical thinking with regard to what we want students to learn, the design process will remain intuitive and unsystematized.

(To be developed: DG seems to be on the verge of using activity theory in a way that confounds ONT1 and ONT2, as discussed earlier.)

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