Composition of Functions

Part 1: An Example

1. There are 3 scales used in measuring temperature – Fahrenheit, Celsius and Kelvin. One can convert Fahrenheit reading to Celsius using $c(t) = \frac{5}{9}(t - 32)$, where t represents temperature in degrees Fahrenheit. Likewise, Celsius readings can be converted to Kelvin using $k(t) = t + 273$, where t represents temperature in degrees Celsius.

   a. I want to convert 100° F to the Kelvin scale. Make the conversion and explain your process.

   b. Convert $-5°$ F to Kelvin

   c. Convert 115° F to Kelvin

   d. Generalize this process by creating a function that will take measurements in degrees Fahrenheit and convert them into degrees Kelvin, and explain how you did that.

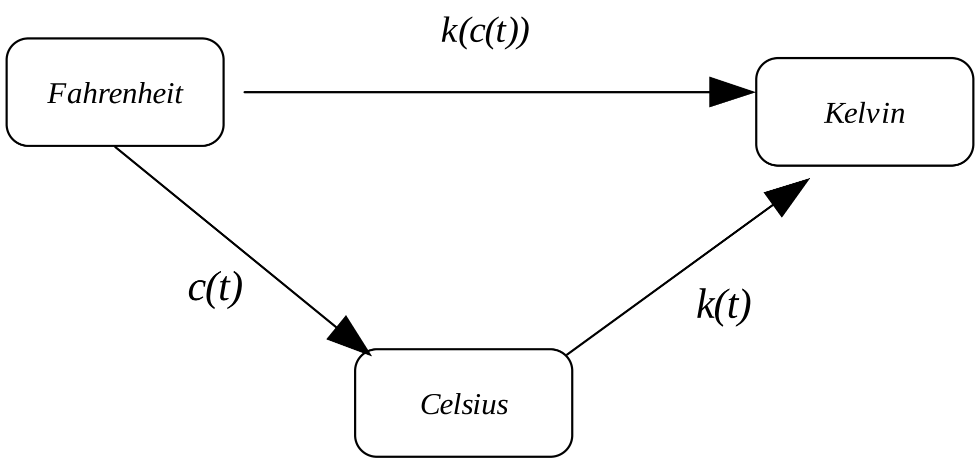
Model #1

Function Chains

$$t \xrightarrow{c} c(t) \xrightarrow{k} k(c(t))$$

Model #2

Change of Variable



Model #3

Modified T-table

Input	Output→Input	Output
t	$c(t)$	$k(c(t))$
Fahrenheit	Celsius	Kelvin

Model #4?

Part 2: Some Big Ideas about Function Composition

1. In mathematics, combining simple functions with "composition" to form more complicated functions is somewhat similar to the way that in chemistry elements can be combined through chemical bonds to form compounds.

2. Function composition combines a pair of functions *f* and *g* into a single "composite" function written $f \circ g$. The composite function operates in a chain where the output of function *g* is used as the input of function *f*.

3. Sometimes the order of the composition of two functions makes a difference, while at other times it doesn't. That is, for some functions *f* and *g*, $f \circ g = g \circ f$ while for other functions *f* and *g*, $f \circ g \neq g \circ f$

4. There are three common operations by which two functions f and g can be combined to form another function *h*: addition $(f + g)$, multiplication $(f \cdot g)$, and composition $f \circ g$. The three are quite different, and function composition needs to be clearly distinguished from the other two.

5. Given a graph of a function *f*, graphs of $f(x+2)$ and $f(x)+2$ represent translations of this graph, while graphs of $f(2x)$ and $2f(x)$ represent expansions or contractions of this graph. These transformations of graphs can be expressed in terms of the composition of *f*.

6. Iteration (repeating a process over and over) can be expressed in terms of the composition of a function with itself.