

**Box Problem Extension**

Complete one of the following extensions to the Box Problem.

*Note: This work should be complete, well organized, and should include a mathematical or scientific conclusion.*

**Extension #1 (Science Application)**

The density of a substance is defined as the ratio of the mass and its volume.

- A. Suppose you have a substance with the property that it conforms to the size of any container while maintaining a constant mass. [*Hint: For the purposes of this investigation, assign the mass a number, such as 1.*] Investigate the changes in the density of the substance as it is contained in all the possible boxes formed from the  $8\frac{1}{2} \times 11$  sheet of material. In your investigation answer the following:
- What function models this situation? Sketch the graph.
  - What is the domain of this function?
  - What is the range of the function?
  - What critical points (maxima or minima) does this function have and how do they relate to the original function associated with the volume of the box?
  - What does this tell you about the substance in question?
  - Does such a substance exist?
- B. Suppose the substance had a fixed density. [*Hint: For the purposes of this investigation, assign the density a number, such as 1.*] Investigate the changes in the mass of the substance as it is contained in all the possible boxes formed from the  $8\frac{1}{2} \times 11$  sheet of material. In your investigation answer the following:
- What function models this situation? Sketch the graph.
  - What is the domain of this function?
  - What is the range of the function?
  - What critical points (maxima or minima) does this function have and how do they relate to the original function associated with the volume of the box?
  - What does this tell you about the substance in question?
  - Does such a substance exist?

**Extension #2 (Uses Calculus)**

- A. Show that when the box formed by the  $8\frac{1}{2} \times 11$  sheet attains the maximum volume, the area of the base is equal to the area of the lateral sides.
- B. Will this occur for any sized rectangular sheet of material? Make a conjecture and prove it.

**Suggestion:**

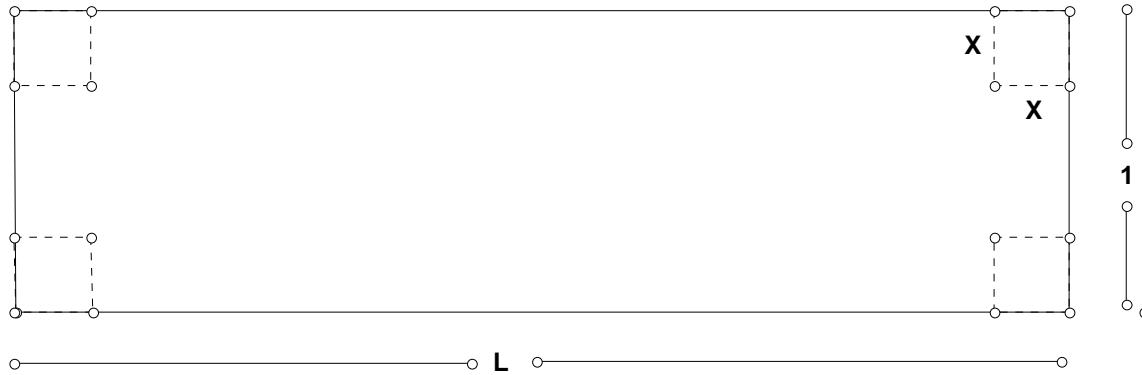
If you designate the rectangular paper as having length **L** and width **1** with square cut out with side **x**, you can account for all possible dimensions with fewer parameters.



### Extension #3 (No Calculus needed)

Suppose the sheet of material was some other size? In this investigation, you will explore the relationship between the maximum volume and the length of the cut out.

If you designate the rectangular paper as having length  $L$  and width  $1$  with square cut out with side  $x$ , you can account for all possible dimensions with fewer parameters.



A. Complete the following table:

| Length | Width | $x$ which maximizes volume | Maximum volume |
|--------|-------|----------------------------|----------------|
| 1      | 1     |                            |                |
| 2      | 1     |                            |                |
| 5      | 1     |                            |                |
| 10     | 1     |                            |                |
| 50     | 1     |                            |                |
| 500    | 1     |                            |                |
| 5000   | 1     |                            |                |

B. Write a paragraph summarizing your conclusions from this investigation. Your summary should include statements about what happens to the domain and range as the length increases, the behavior of  $x$  as the length increases, and any other patterns you discover.

C. Solve the following problem:

Ms. Newton wishes to fence off a rectangular region by her house such that three sides of the region will be fenced and the house will form the other side. She buys a length of fencing at Ace Hardware. What dimensions will yield the largest possible area if we consider the fencing to be of length equal to 1 unit?

D. Use your solution to the problem in C) to explain your results in B).