

Making sense of inverse functions through stacks of cups

Part A: Start with a sleeve of Styrofoam cups.

1. Create a table of data by measuring the height of the stack for each particular number of cups.

n	h in cm
1	
3	
6	
7	
10	
12	
14	

S = the 'stick-up' height of a cup
 B = the individual cup height minus the stick-up part

2. Determine the height of a stack of 40 cups is. Explain how you determined your solution and why you selected that strategy.
3. Create a graph on a sheet of graph paper that represents the height, h , as a function of the number, n , of cups, for $n = 1$ to $n = 20$. Label the axes
4. Find the domain and range of this function and describe the inputs and outputs in the context of the problem
5. Name this function H and write an algebraic formula that expresses the height of the stack as a function of the number of cups.

Part B

6. You have been asked to design a machine to move the cups and need to be able to easily compute the number of cups that can fit into a space of h inches. We wish to construct a new function that outputs (or computes) the number of cups, n , for a given input of a stack of height h . On a clean sheet of graph paper, draw a graph to represent this function for reasonable values of h between $h = 1$ and $h = 30$ cm. Label the axes, and explain how you determined the graph of this function.
7. Write an algebraic representation of this function N and find the domain and range of N .

8. Compare and contrast H (the height function) and N (the number-of-cups function) in their various representations. (For example, compare the graphs, compare the tables, etc.) How are H and N related in terms of the context of stacking cups?

Part C

9. Mathematically, we would say that H and N are *inverses* of each other. What does it mean for one function to be an inverse of the other?
10. In algebra, students are often taught to find the inverse of a function by “flipping” the graph over the line $y = x$. Make sense of this statement. Explain what it means and determine if this is an accurate “tool” in this context. Include all three representations (table, graphs, and formulas) in your response.
11. Sometimes students are taught to find the inverse of a function by “exchanging x and y ”. Make sense of this statement. Explain what it means and determine if this is an accurate “tool” in this context. Include all three representations (table, graphs, and formulas) in your response.

Part D

You will need a pair of transparencies to complete this section. If you do not have access to any, let your instructor know.

12. Using two transparencies, trace the graphs of H and N (one per transparency). If your graphs of the height function and the number of cups function used different scales, re-draw the graphs, with the axes of the two graphs having the same scales. Overlap these graphs. What are your observations?
13. ‘Overlapping the graphs’ is equivalent to drawing the two graphs on the same set of axes. Does it make sense to graph H and N on the same set of axes in this way? Take a moment to revisit your response to question #10. Does it make sense to find the inverse by “flipping” the graph over the line $y = x$? Why or why not?
14. Consider graphing the two functions on the standard x, y - coordinate axes, where x represents the input of the function, and y represents the output, and both x and y are unitless numbers without reference to the physical realities these functions represent. How does this change the meaning/interpretation of the graph?